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# Power Corrections in QCD: A Matter of Energy Resolution.

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## Abstract

We consider power-like corrections in QCD which can be viewed as power suppressed infrared singularities. We argue that the presence of these singularities depends crucially on the energy resolution. In case of poor energy resolution, i.e., inclusive cross sections, there are constraints on infrared singularities expressed by the Kinoshita-Lee-Nauenberg (KLN) theorem. We rewrite the theorem in covariant notations and argue that the KLN theorem implies the extension of the Bloch-Nordsieck cancellation of logarithmic singularities to the case of linear corrections.

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The emission of soft particles in gauge theories exhibits remarkable regularities. In particular, the spectrum of infrared photons is universally proportional to  $d\omega/\omega$ . Such a spectrum implies a logarithmic divergence upon integration over the photon energy  $\omega$ . Moreover, the divergence is cancelled if the emission cross section is summed up with radiative corrections to the process without a soft photon; the famous Bloch-Nordsieck cancellation [1]. As a result the physical cross section can contain only the log of the physical energy resolution,  $\Delta E$ .

The general nature of this cancellation is revealed by a quantum-mechanical theorem, due to Kinoshita, Lee and Nauenberg (KLN) [2], according to which all infrared singularities are canceled provided that summation over all degenerate in energy states is performed. An important point is that summation over both initial and final states is required:

$$\sum_{i,f} |S_{i \rightarrow f}|^2 \sim \text{free of singularities} \quad (1)$$

where  $S_{i \rightarrow f}$  are the elements of the  $S$ -matrix. Since the summation over initial states does not correspond to an experimental resolution, infrared singularities persist, generally speaking, in physical cross sections. From this point of view the Bloch-Nordsieck cancellation, upon the summation over final states alone, looks as an exception rather than a rule. The reason for this exception is that in the limit of vanishing photon energy, the emission and absorption of a photon are indistinguishable.

In QCD, the problem of infrared singularities is compounded due to the essential presence of collinear singularities, and as such, infrared sensitive quantities are affected by confinement and cannot be calculated reliably. One is therefore led to consider observables that are infrared safe or those in which the long distance effects can be isolated into a universal factor [3]. In the latter approach, the effect of the collinear singularities is then embedded into phenomenological structure functions  $f_a^h(x)$  where  $x$  is the momentum fraction of the parent hadron  $h$  carried by parton  $a$ . In particular, the cross section for the Drell-Yan (DY)

process  $h_1 + h_2 \rightarrow \mu^+ \mu^- + X$  is given by [3]:

$$\frac{d\sigma}{dQ^2}(\tau, Q^2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx \delta(\tau - x_1 x_2 x) f_a^{h_1}(x_1) f_b^{h_2}(x_2) (\sigma_0 W_{ab}(x, Q^2)) \quad (2)$$

where,

$$\sigma_0 = \tau \frac{4\pi\alpha_{QED}^2}{9Q^4}. \quad (3)$$

$S, Q^2$  are squares of the hadronic and leptonic invariant masses, respectively,  $\tau = Q^2/S$  and  $W_{ab}(x, Q^2)$  is the appropriately normalized (hard) cross section for  $a + b \rightarrow \mu^+ \mu^- + \text{partons}$ , and  $x = Q^2/s$  with  $s$  the invariant mass squared of the partons  $a, b$ . The structure functions can be deduced from another process, say, deep inelastic scattering. As an example of the first approach, one is led to the construction of infrared safe variables such as thrust  $T$  in  $e^+e^-$  annihilation:

$$T = \max_{\mathbf{n}} \frac{\Sigma(\mathbf{p}_i \mathbf{n})}{\Sigma|\mathbf{p}_i|}. \quad (4)$$

where  $\mathbf{p}_i$  are the momenta of particles while  $\mathbf{n}$  is a unit vector.

Most recently, the presence of linear terms,  $\sim \Lambda_{QCD}/Q$  where  $Q$  is a large mass parameter, has attracted attention [4, 5, 6, 7]. These terms do not jeopardize the calculability of various observables but provide us with a measure of their infrared sensitivity. Consider, for example, the emission of a soft gluon in the case of thrust:

$$(1 - T)_{soft} \sim \int_0^{\Lambda_{QCD}} \frac{d\omega}{\omega} \frac{\omega}{Q} \alpha_s(\Lambda_{QCD}^2) \sim \frac{\Lambda_{QCD}}{Q} \quad (5)$$

where  $\Lambda_{QCD}$  is an infrared cut off such that  $\alpha_s(\Lambda_{QCD}) \sim 1$ ,  $Q$  is the total energy and the factor  $\omega/Q$  is due to the definition of the thrust (4). Eq (5) clearly demonstrates the presence of linear terms in thrust and that they arise due to soft gluons. This is a general feature [6, 7]. Remarkably enough, recent developments indicate a universality of these terms to all orders in the large coupling  $\alpha_s(\Lambda_{QCD})$  [6, 7], elevating their status to that of the logarithmic divergencies. The statement on the universality of linear corrections is formulated, however, in less transparent terms. Two ingredients are essential: the renormalon technique [8] and

use of resummed cross sections [4, 7]. In particular, in case of thrust one can show [7] that

$$\langle e^{-\nu(1-T)} \rangle_{1/Q} = e^{-\nu E_{soft}} \quad (6)$$

where  $\nu$  is a (large) parameter and the universal quantity  $E_{soft}$  is expressed in terms of the cusp anomalous dimension  $\gamma_{eik}$ :

$$E_{soft} = \frac{1}{Q} \int \frac{dk_{\perp}^2}{k_{\perp}^2} \gamma_{eik}(\alpha_s(k_{\perp})) k_{\perp}. \quad (7)$$

The anomalous dimension  $\gamma_{eik}$  controls various hard cross sections and is calculable perturbatively [10]. Moreover, a similar  $1/Q$  piece can be identified in the DY cross section [4, 6, 7]. Thus, linear terms appear to be no less universal than the leading log corrections, the technical complications being due to the non-abelian nature of gluons.

In this note we will present arguments that linear terms and soft logarithmic divergencies share not only the property of universality but in some cases, a Bloch-Nordsieck type of cancellation as well. Namely, if one considers an inclusive cross section, that is, a case of poor energy resolution, the linear terms cancel. If, on the other hand, the accuracy of measurements on the final state is of the order of an infrared parameter, then linear terms survive. As an example of an inclusive cross section we will consider the DY process. Observables which assume precision measurements are exemplified by thrust. We have explicitly verified our arguments for one- and two-loop abelian gluons and details will be presented elsewhere [15]. We believe that our strategy and techniques can be generalized to a formal all orders proof. Our search for a Bloch-Nordsieck type of cancellation was stimulated in fact by the results of Ref. [11] where it was shown that if one uses a photon mass  $\mu$  as an infrared regulator then linear in  $\mu$  terms cancel from the DY cross section at the one-loop level. We are establishing a general principle behind this apparently accidental one-loop cancellation. As for observables which assume precision measurements like thrust, the presence of  $1/Q$  terms in these has been confirmed and the KLN based arguments presented below are inapplicable.

Thus, we will next consider the  $1/Q$  corrections to the DY cross section for large  $\tau$ , ( $\tau \rightarrow 1$ ), for which the dominant partonic process is  $q\bar{q}$  annihilation and in which the emitted gluons

are soft with their energy bounded by  $Q(1 - \tau)$ . Our analysis hinges crucially on the KLN theorem. To be precise, consider the quantity:

$$P_{mn} = \frac{1}{m! n!} \sum_{i,f} |M_{mn}|^2 \quad (8)$$

where  $M_{mn}$  is the amplitude for the process  $q + \bar{q} + m \text{ gluons} \rightarrow \gamma^* + n \text{ gluons}$ . In general,  $P_{mn}$  contains contributions from disconnected diagrams of  $M_{mn}$ . We will refer to  $P_{mn}$  as the Lee-Nauenberg probabilities. The assertion of the KLN theorem is that the quantity  $P$ ,

$$P = \sum_{m,n} P_{mn} \quad (9)$$

contains no infrared sensitivity. In particular, and what is relevant for our discussion, it does not contain terms linear in an infrared cut off  $\lambda$  (which among other possibilities, can be  $(k_\perp)_{min}$ , or the mass of a U(1) gauge boson).

Keeping in mind that the linear terms originate due to soft radiation, let us consider the case of single soft gluon emission in the DY process. We can have, therefore, the diagrams with the soft gluon in the final state, see Fig. 1(b). Arrows on the gauge boson line denote the direction of momentum flow and we have also indicated the unitarity cut. The open circles denote the interaction with the  $\gamma^*$  which is not explicitly considered. The set of all such diagrams will be denoted by  $P_{01}$ . Moreover, according to the KLN prescription for degenerate states, we must consider diagrams where the soft gluon is absorbed in the initial state. Thus we are led to consider in  $P_{mn}$  of eq (8) absorption amplitudes squared which correspond to the diagrams in Fig. 1(c). We denote the set of all such diagrams by  $P_{10}$ . Note that superficially,  $P_{10}$  and  $P_{01}$  look similar, however, the energy momentum constraints are different for the two sets. Finally, we have diagrams with virtual gluons alone. In fact, we can include the hard interactions as well as virtual gluons into a single blob and we will follow this notation (hatched region in the figures). Thus the diagram in Fig. 1(a) will be denoted by  $P_{00}$  and includes the contribution of the virtual gluons.

In addition to the above, we also have the contribution of the disconnected diagrams.

From the interference between the connected and disconnected diagrams , we can get contributions to  $P_{mn}$  from sets of the type in Figs.1(d), 1(e) . These sets will be denoted generically by  $P_{11}^{(1)}$ . Notice that the quark and antiquark propagators are the same in  $P_{00}$  as in the corresponding  $P_{11}^{(1)}$ . In fact it is easy to check that  $P_{11}^{(1)}$  can be obtained from  $P_{00}$  by the replacement of the propagator of the virtual gluon by  $-2\pi\delta(k^2)$ . The contribution from disconnected diagrams do not end here. In fact an infinite number of disconnected gluon lines can be added without changing the order of perturbation theory. An analysis shows, and this is one of our main results, that after a suitable rearrangement of the perturbation series, the contribution of the disconnected pieces can be multiplicatively factored out:

$$P = \sum_{m,n} P_{mn} = \sum_{m,n} \sum_{\alpha} \left( \frac{1}{(m-\alpha)! (n-\alpha)!} D_d(m-\alpha, n-\alpha) \right) \cdot (P_{00} + P_{01} + P_{10} + \tilde{P}_{00}). \quad (10)$$

Here,  $D_d(m-\alpha, n-\alpha)$  denotes the disconnected Green's function with  $(m-\alpha)$  ingoing and  $(n-\alpha)$  outgoing gluons. Moreover,  $\tilde{P}_{00}$  is identical to  $P_{00}$  in all respect but one. Namely, the gluon propagator of  $P_{00}$  is replaced by its complex conjugate:

$$-2\pi\delta(k^2) + \frac{i}{k^2 + i\epsilon} = \frac{i}{k^2 - i\epsilon} = \left( \frac{-i}{k^2 + i\epsilon} \right)^*. \quad (11)$$

Thus,  $\tilde{P}_{00}$  cannot be evaluated by applying the standard Feynman rules.

Now, from the KLN theorem we know that the sum  $(P_{00} + P_{01} + P_{10} + \tilde{P}_{00})$  does not contain any IR sensitive terms. Note that the physical Bloch-Nordsieck crosssection corresponds to the sum  $P_{00} + P_{01}$ . Let us first briefly consider the soft logarithmic divergences. It is easy to see that in the leading order as the gluon energy vanishes, then for the log divergent terms,  $P_{01} \sim P_{10}$  and for the virtual corrections to this accuracy we may replace the gluon propagator in both  $P_{00}$  and  $\tilde{P}_{00}$  by the  $\delta(k^2)$  piece. Thus from the KLN theorem we conclude that  $P_{00} + P_{01}$  is free of soft logarithmic divergences, which is just the statement of the Bloch-Nordsieck cancellation at this accuracy. We would like to extend this argument to the linear terms in the DY crosssection as well.

Specifically, we identify the soft component of  $\sigma_0 W_{q\bar{q}}(x)$  in eq.(2) for large  $\tau$  with the inclusive probabilities  $P_{00} + P_{01}$  above. Then we take the moments with respect to  $\tau$  of eq.(2), upon which, it reduces to a product of moments of the structure functions and of  $\sigma_0 W_{q\bar{q}}$ . The moments of this latter quantity are with respect to  $x$ , which in turn is related to the energy of the emitted gluon in the soft region (large moments) by [9]

$$\omega = Q(1 - x)/2. \quad (12)$$

If, for example, we denote the transverse momentum cutoff of the emitted gluon by  $\lambda$  then  $x$  is bounded by  $(1 - 2\lambda/Q)$ . Thus, linear terms in the cut-off do not arise from the virtual corrections in DY. Then we see from eq(10) that the quantity  $P_{01} + P_{10}$  does not contain terms linear in  $\lambda$  (upon taking moments, which is henceforth understood).

We will next show that it follows from the Low theorem [12] that

$$P_{01} - P_{10} = O(\lambda^2) + \text{finite terms independent of } \lambda. \quad (13)$$

Then since the virtual gluon diagrams cannot contribute linear terms in  $\lambda$  it follows from the above that  $P_{01}$  has infrared sensitivity of  $O(\lambda^2)$ .

We can show using the Low theorem that to linear accuracy:

$$P_{01} = \left( \sum |T|^2 \right) \frac{2\alpha_s}{\pi} \int \frac{d^3k}{2k_0} \delta((p_1 + p_2 - k)^2 - Q^2) \cdot \frac{2Q^2 s}{2(p_1 \cdot k)2(p_2 \cdot k)} \quad (14)$$

where,  $p_1, p_2$  are the momenta of the quark (antiquark) respectively and  $T$  is the radiationless amplitude, i.e., for the process  $q + \bar{q} \rightarrow \gamma^* \rightarrow l^+ l^-$ . To see this, we note that the amplitude  $M_{01}$  can be expressed upto terms  $\omega^0$  as:

$$M_{01}^\alpha = g_s \frac{(2p_2 - k)_\alpha}{2p_2 \cdot k - k^2} T - g_s \frac{(2p_1 - k)_\alpha}{2p_1 \cdot k - k^2} T + g_s (p_1 + p_2) \cdot k \left( \frac{p_{1\alpha}}{p_1 \cdot k} - \frac{p_{2\alpha}}{p_2 \cdot k} \right) \frac{\partial T}{\partial s} \quad (15)$$

where,  $s \equiv p_1 \cdot p_2$  and  $g_s^2/(4\pi) = \alpha_s$ . Next, in calculating  $\sum |M_{01}|^2$  we keep terms  $O(1/k^2)$  and  $O(1/k)$  and in this way arrive at the expression given above. Similar expressions can be obtained for  $P_{10}$  and we find their moments to be equal for a fixed  $Q^2$ , to this accuracy.

Thus, we conclude that the DY cross section, has no terms linear in the infrared cut off  $\lambda$ . The universal term is cancelled from the inclusive cross section as was observed first by an explicit calculation in Ref [11].

The approach of the present paper can be generalized to higher orders. We have checked explicitly [15] that at least to two loops in a theory with abelian gluons all steps indeed generalize. In particular, the observation (10) that the KLN sum (8) involves in fact both standard and complex conjugated gluon propagators remains true and evolves into a pattern which can be readily generalized to any order. Thus, the expression for the probability  $P$  for the case of two gluons again assumes a form in which the disconnected pieces may be factorized out, i.e.,

$$P = \sum_{m,n} P_{mn} = \sum_{m,n} \sum_{\alpha} \left( \frac{1}{(m-\alpha)! (n-\alpha)!} (\alpha+1) D_d(m-\alpha, n-\alpha) \right) \cdot \left( P_{00} + P_{01} + P_{02} + P_{10} + P_{20} + \tilde{P}_{00} + P_{11}^{(2)} + \tilde{P}_{10} + \tilde{P}_{01} + \tilde{P}_{00} \right), \quad (16)$$

where, to this order,  $P_{01}$  contains the square of the amplitude with one virtual and one real emitted gluon, and  $\tilde{P}_{01}$  is obtained from the former by symmetrically replacing the virtual gluon line in the amplitude by its complex conjugate. Similarly for  $P_{10}$  and  $\tilde{P}_{10}$  which apply for the absorbed gluon.  $\tilde{P}_{00}$  is obtained from  $P_{00}$  by replacing both the virtual gluon lines in the corresponding amplitude by their complex conjugates, whereas  $\tilde{P}_{00}$  is obtained by symmetrically replacing only one.  $P_{02}$  refers to the probability for the emission of two gluons and  $P_{20}$  to the corresponding absorption amplitude.  $P_{11}^{(2)}$  is the probability for the emission of one gluon and the absorption of another. Next the Low theorem may be generalized to the case of two abelian soft gluons and this can be used to relate the corresponding emission and absorption probabilities to linear accuracy. Proceeding in a manner analogous to the one gluon case, we can then show again that the KLN theorem implies the Bloch-Nordsieck cancellation of terms both logarithmic and linear in  $\lambda$ . Moreover, at least in the theory with abelian gluons, the factorization of the linear terms as well, in the inclusive probabilities is manifested [14]. Details will be given elsewhere [15].

To summarize, we have suggested a criteria for the presence of  $1/Q$  corrections in observables. For those having poor energy resolution we have presented arguments that combining the KLN and Low theorems provides a powerful means of proving a Bloch-Nordsieck type of cancellation valid to linear approximation in an infrared cut off. Let us also mention that the KLN theorem allows for a compact derivation of the cancellation of power-like corrections in inclusive weak decays found explicitly in Ref [13] in the one loop order. Moreover, the generalization to higher loops is straightforward [15].

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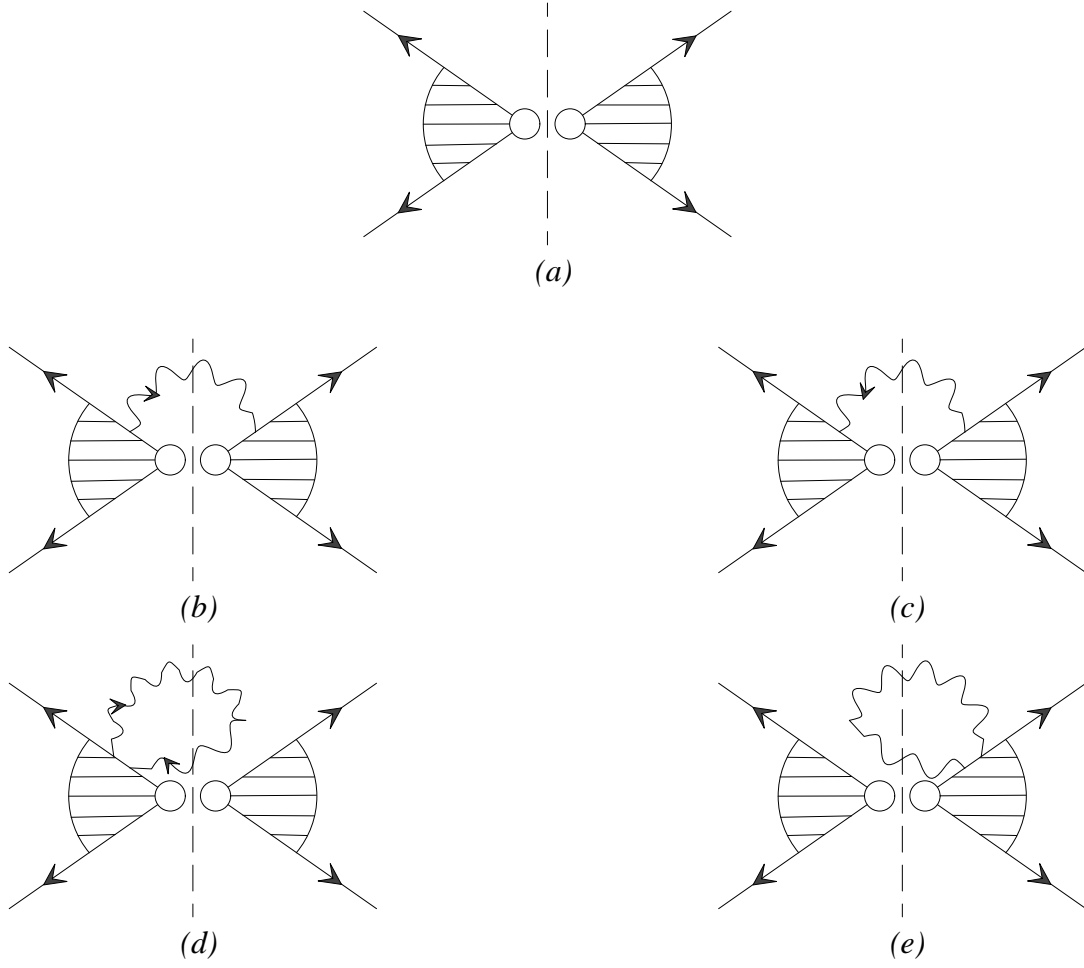


Figure 1: Cut diagrams representing (a)  $P_{00}$ , (b)  $P_{01}$ , (c)  $P_{10}$ , and (d) + (e)  $P_{11}^{(1)}$ .